

# INTRODUCTION TO CONIC SECTIONS

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Math 5

# Historical Background

- **Menaechmus (350 B.C.)**
  - tutor to Alexander the Great
  - credited with the discovery of conic sections around 360-350 B.C.
  - in an attempt to solve the three famous construction problems of **trisecting the angle**, **doubling the cube**, and **squaring the circle**
- **Aristaeus (310 B.C.)**
  - first to treat conics as loci or set of points

<http://www.math.rutgers.edu/~cherlin/History/Papers1999/schmarge.html>

[http://usiweb.usi.edu/students/gradstudents/j\\_k\\_l/kleinknecht\\_s/portfolio/Educ%20690\\_004%20ST/History%20of%20Conics.htm](http://usiweb.usi.edu/students/gradstudents/j_k_l/kleinknecht_s/portfolio/Educ%20690_004%20ST/History%20of%20Conics.htm)

# Historical Background

- **Euclid (310 B.C.)**
  - wrote 4 books on “Conics”
  - compiled all works on conics up to his time
- **Archimedes (287 B.C.)**
  - used extensive knowledge of conics to solve famous geometric problems of that time

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[http://usiweb.usi.edu/students/gradstudents/i\\_k\\_l/kleinknecht\\_s/portfolio/Educ%20690\\_004%20ST/History%20of%20Conics.htm](http://usiweb.usi.edu/students/gradstudents/i_k_l/kleinknecht_s/portfolio/Educ%20690_004%20ST/History%20of%20Conics.htm)

# Historical Background

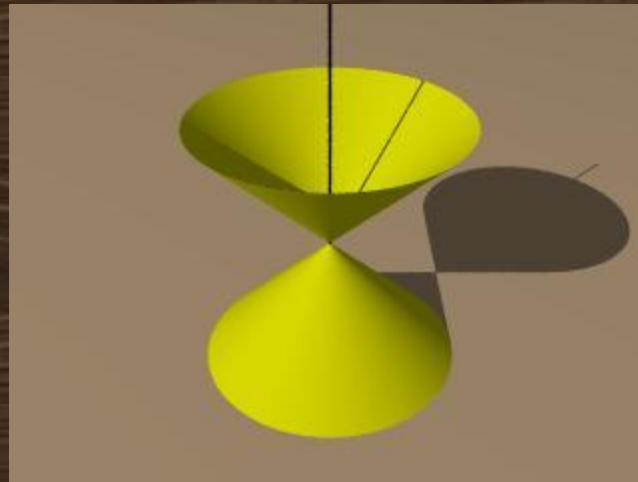
- **Apollonius (262 B.C.)**
  - published 8 books on conic sections
  - including the 1<sup>st</sup> 4 which are compilations of the previous works on conics
  - provided most of the common terms used in conics today
  - clearly did the most extensive study on conics

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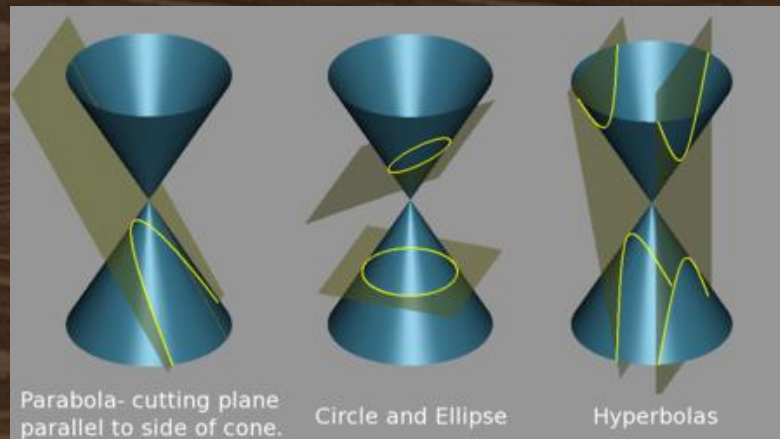
# Geometric Properties

- CONE:
  - a 3-D surface resembling 2 ice cream cones facing opposite directions and sharing the same apex/vertex
  - generated by rotating a diagonal line (generator) with respect to a vertical line (axis)
  - consists of two nappes, the upper and the lower



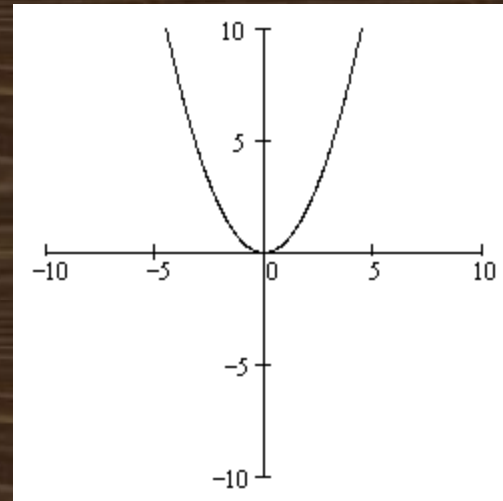
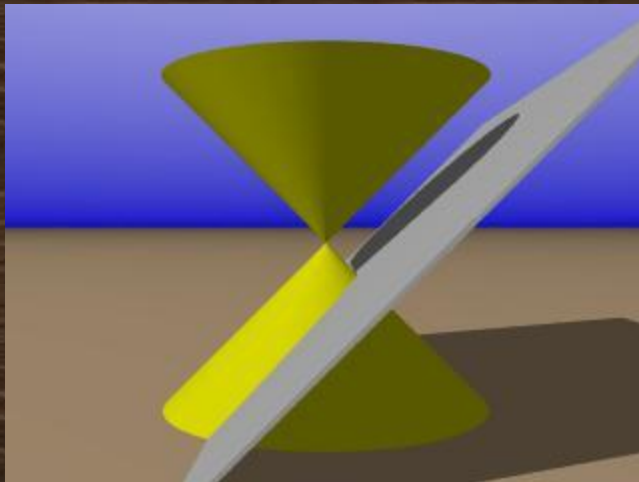
# Geometric Properties

- CONIC SECTION:
  - curve of intersection of a plane with a (right circular) cone
  - type of conic depends on inclination of the plane
  - general case produces parabola, ellipse, or hyperbola
  - degenerate case (plane passes through apex) produces point, line, or 2 intersecting lines



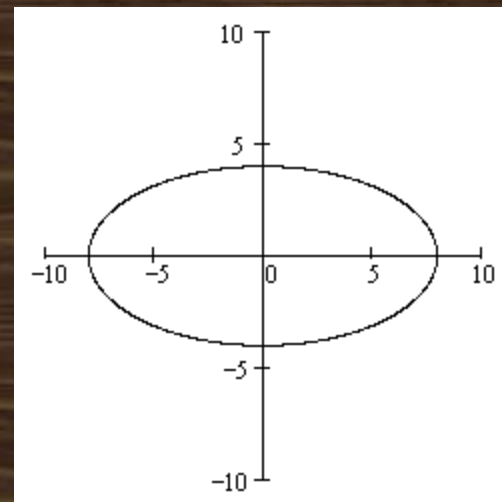
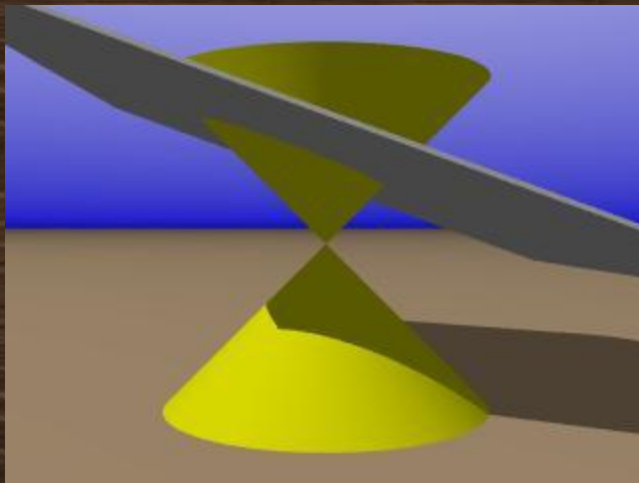
# Geometric Properties

- PARABOLA:
  - cutting plane is parallel to 1 generator
  - figure consists of 1 open curve



# Geometric Properties

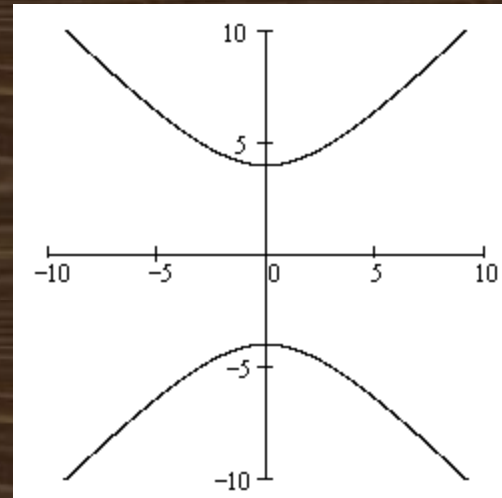
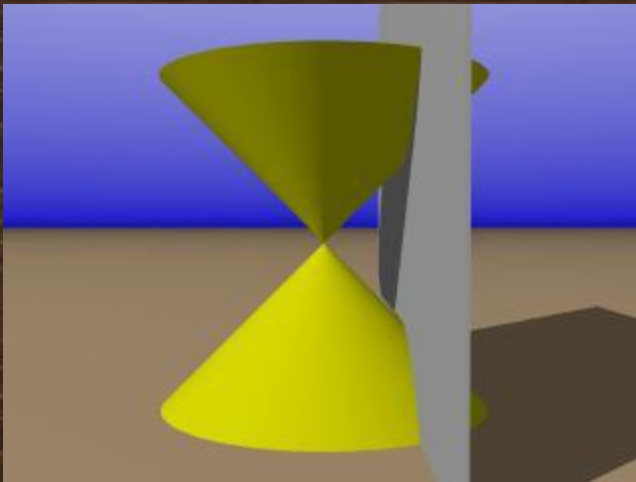
- ELLIPSE:
  - cutting plane intersects all generators
  - figure consists of 1 closed curve





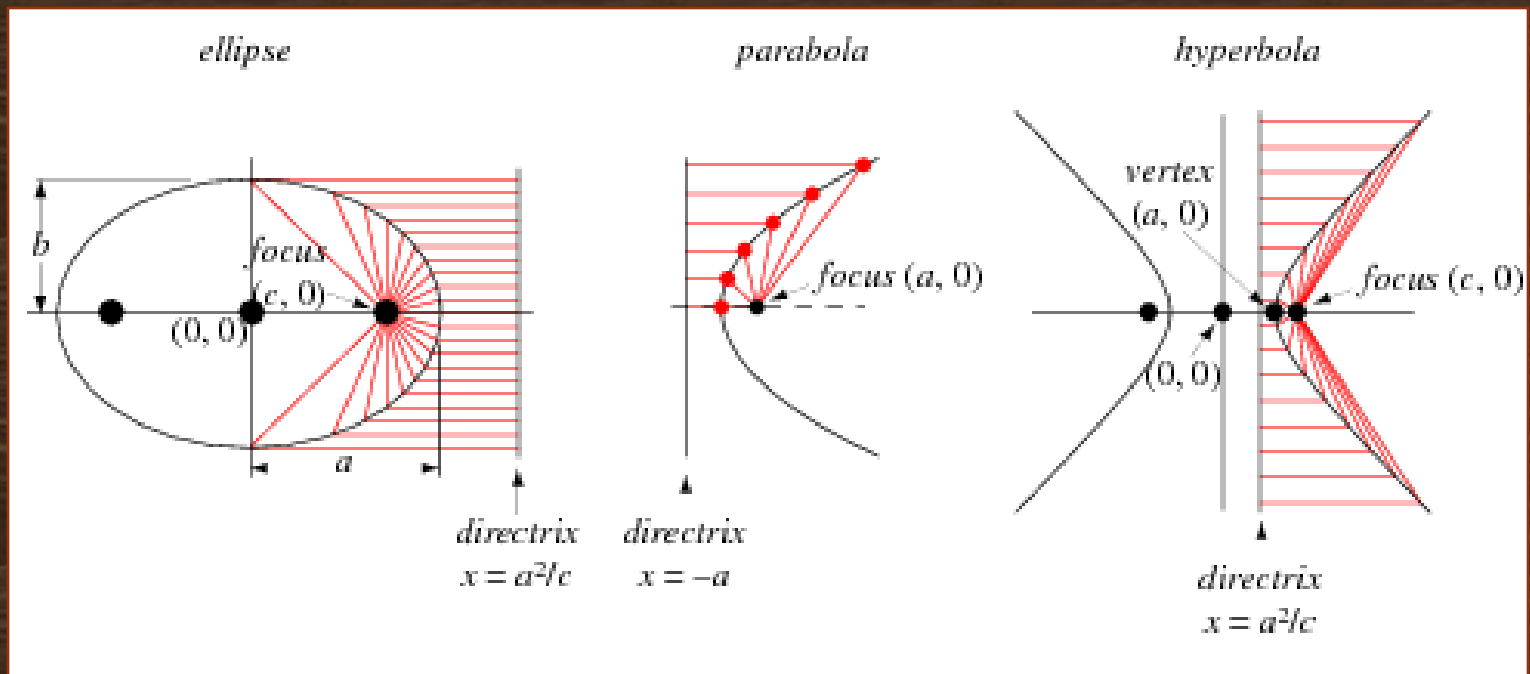
# Geometric Properties

- HYPERBOLA:
  - cutting plane is parallel to 2 generators
  - figure consists of 2 open curves



# Analytic Properties

- CONIC SECTION:
  - set of points in a plane the ratio of whose distances from a fixed point to its distance from a fixed line is a constant



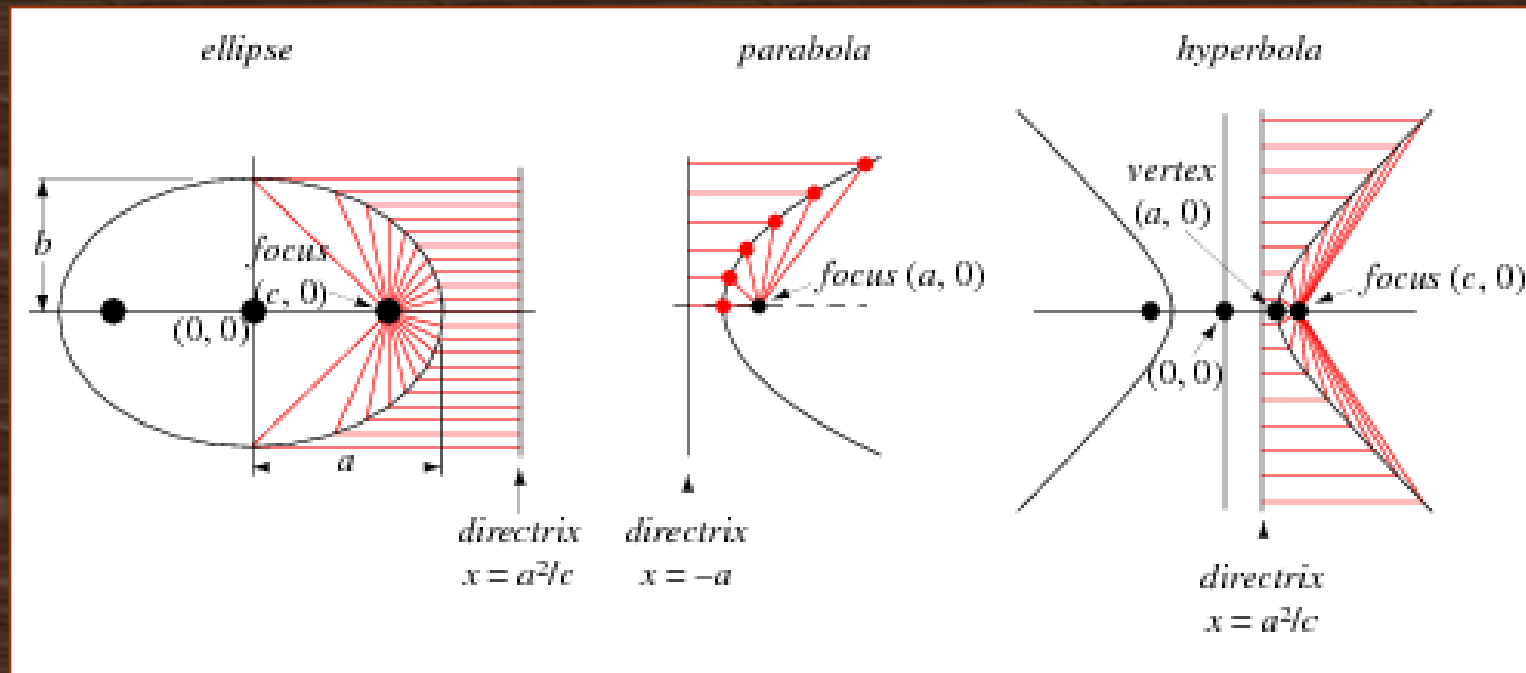
# Analytic Properties

- CONIC SECTION:
  - constant ratio is called eccentricity (denoted by  $e$ )
  - fixed point is called focus (denoted by  $F$ )
  - fixed line is called directrix (denoted by  $D$ )

$$\frac{PF}{PD} = e$$

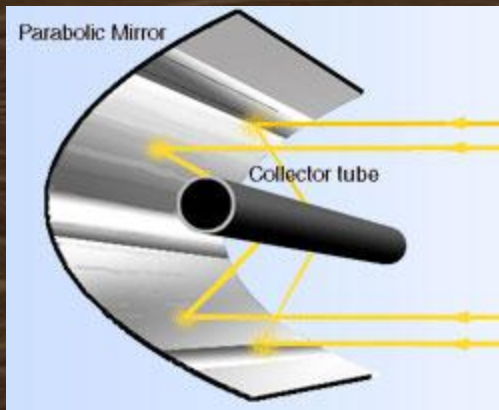
# Analytic Properties

- CONIC SECTION:
  - if  $e < 1$ , the conic is an ellipse
  - if  $e = 1$ , the conic is a parabola
  - if  $e > 1$ , the conic is a hyperbola



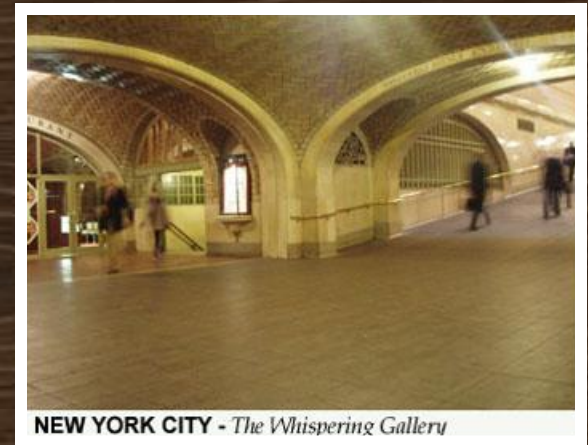
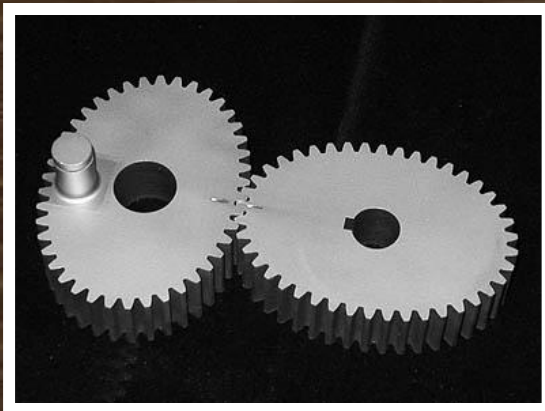
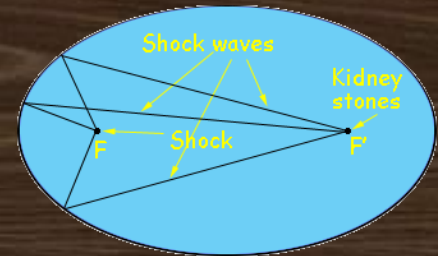
# Applications

- PARABOLA:
  - flashlights, headlights, searchlights
  - satellite dishes, antenna of radio telescope, solar furnace
  - cable of a suspension bridge, arch of an arch bridge
  - path of a projectile



# Applications

- ELLIPSE:
  - astronomy (planetary orbits)
  - whispering galleries (US Capitol, Mormon Tabernacle)
  - roads (elliptical road, QC), pool table, football, bilobe gears
  - lithotripsy (blasting of stones like kidney stones)



# Applications

- HYPERBOLA:
  - LORAN (terrestrial navigation system)
  - design of cooling towers
  - gear transmission (between two skew axes)
  - light from lamp shade
  - sonic boom

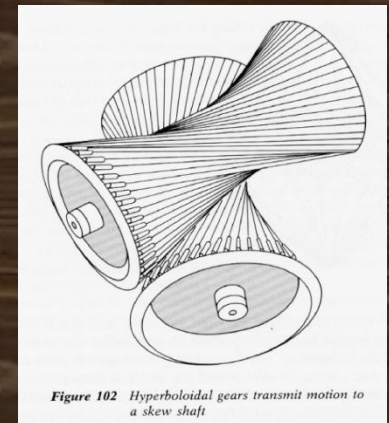
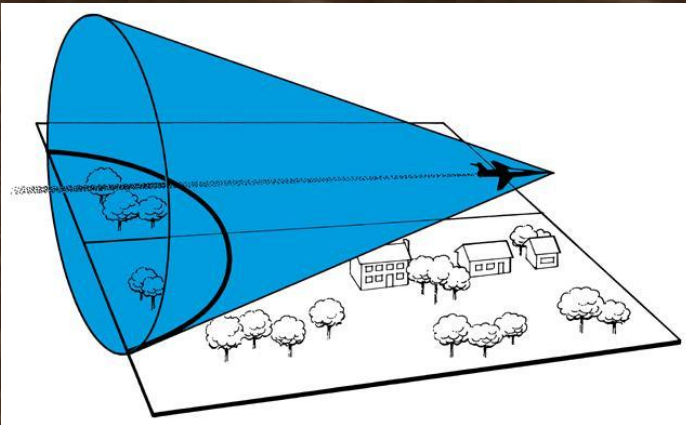


Figure 102 Hyperboloidal gears transmit motion to a skew shaft

# NEXT TOPICS:

- PARABOLA: equations (standard and general) and applications
- TRANSLATION OF AXES

ASSIGNMENT: Read TCWAG6 Section 10.1