Petri Español
MATH 5

PARABOLA AND TRANSLATION OF AXES

OBJECTIVES:

- derive the standard equation of a parabola
- use the equation of a parabola to determine its properties
- find the equation of a parabola given some of its properties
- express the equation of a parabola in both the standard and general forms
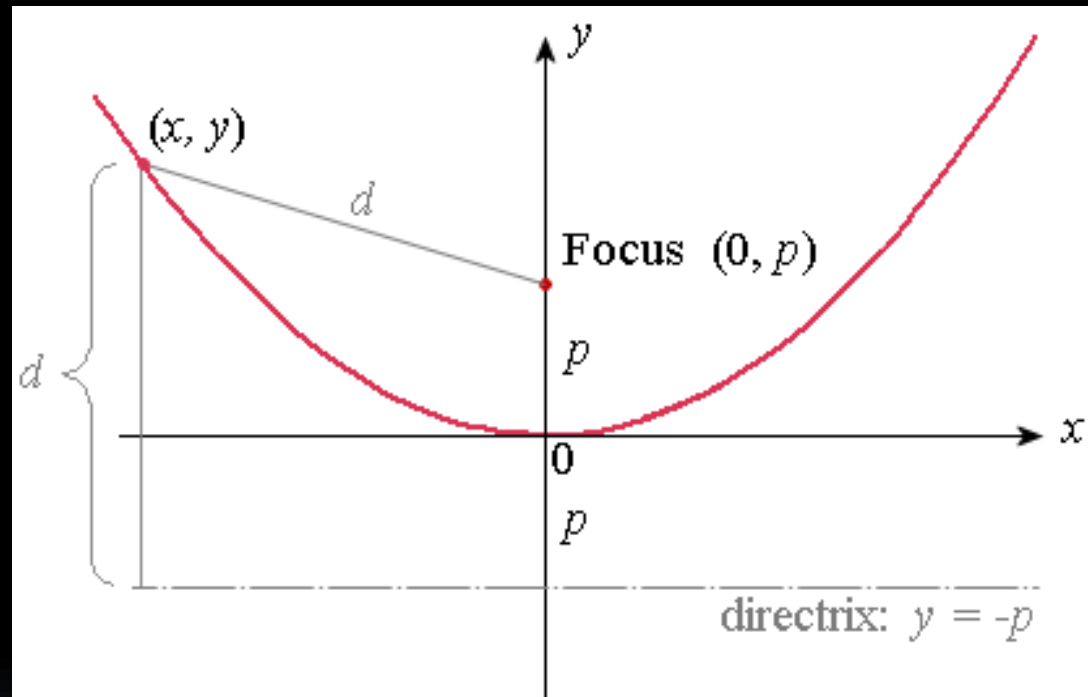- solve problems using equations of parabolas

ANALYTIC DEFINITION

PARABOLA is a set of all points in a plane equidistant from a fixed point (FOCUS) and a fixed line (DIRECTRIX).
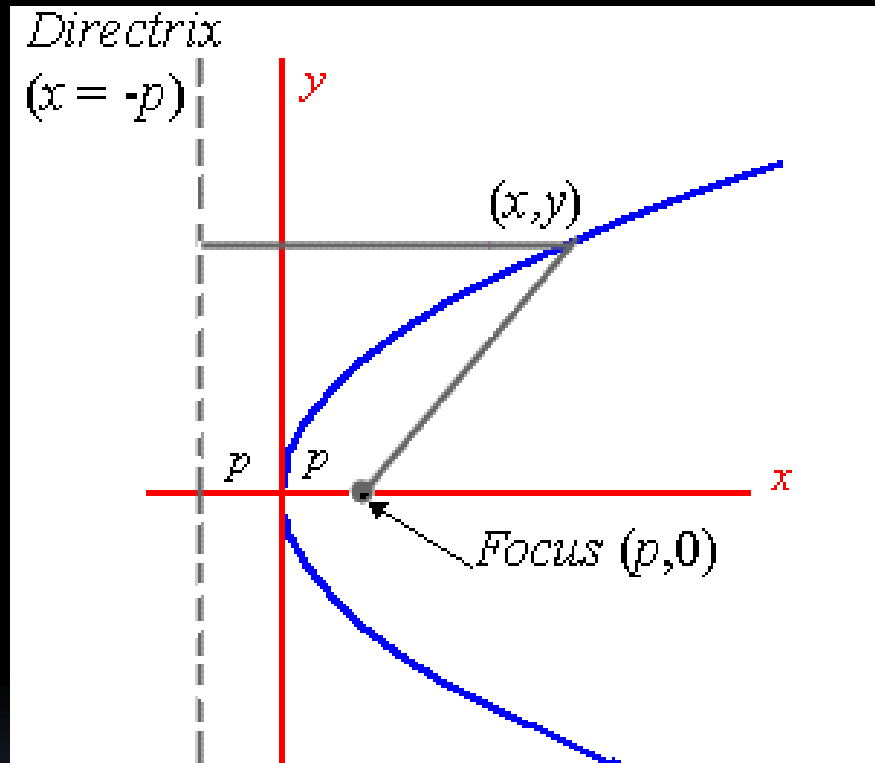
VERTICAL PARABOLA

V(0, 0), F(0, p): x² = 4py

HORIZONTAL PARABOLA

$V(0, 0), F(p, 0): y^2 = 4px$

EXAMPLE 1:

Find the equation of the parabola with vertex (0, 0) and focus (4, 0).

$$y^2 = 16x$$

EXAMPLE 2:

Find the vertex, focus, and directrix of the parabola with equation $x^2 = -2y$.

V(0, 0), F(0, -1/2), D: y = 1/2

EXAMPLE 3:

A satellite dish has a diameter of 1m on its rim and is 10cm deep at the center. How far is its focus from its vertex?

62.5 cm

TRANSLATION OF AXES:

HORIZONTAL TRANSLATION

1. h units to the right

   Replace x by x – h

2. h units to the left

   Replace x by x + h

EXAMPLE 4:

If the graph of $y = x^2$ is translated 3 units to the right, what is the equation of the new graph?

$$y = (x - 3)^2$$

TRANSLATION OF AXES:

VERTICAL TRANSLATION

1. k units up

   Replace y by y – k

2. k units down

   Replace y by y + k

EXAMPLE 5:

If the graph of $y = x^2$ is translated 4 units down, what is the equation of the new graph?

$$y + 4 = x^2 \quad \text{or} \quad y = x^2 - 4$$

EXAMPLE 6:

If the graph of $y = x^2$ is translated 1 unit up and 2 units to the left, what is the equation of the new graph?

$$y - 1 = (x + 2)^2$$

STANDARD EQUATION OF THE PARABOLA WITH VERTEX (h, k):

1. Vertical Parabola

$$(x - h)^2 = 4p(y - k)$$

2. Horizontal Parabola

$$(y - k)^2 = 4p(x - h)$$

EXAMPLE 7:

What is the equation of the parabola with V(3, -2) and F(0, -2)?

$$(y + 2)^2 = -12(x - 3)$$

EXAMPLE 8:

Find the vertex, focus, and directrix of the parabola

$$(x - 2)^2 = 8(y + 1)$$

V(2, -1), F(2, 1), D: y = -3

EXAMPLE 9:

Find the vertex, focus, and directrix of the parabola

$$y^2 + 4x - 6y + 13 = 0$$

V(-1, 3), F(-2, 3), D: x = 0

HOMEWORK#1:

TCWAG6

Section 10.1 Exercises

#s 20, 30, 40, 42