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MATH 5

ELLIPSE

OBJECTIVES:

- derive the standard equation of an ellipse
- use the equation of an ellipse to determine its properties
- find the equation of an ellipse given some of its properties
- express the equation of an ellipse in both the standard and general forms
- solve problems using the equation of an ellipse

ANALYTIC DEFINITION

ELLIPSE: set of all points in a plane the sum of whose distances from two fixed points is a constant.

PARTS OF AN ELLIPSE

FOCI: the two fixed points in the analytic definition, denoted by F.

PARTS OF AN ELLIPSE

PRINCIPAL AXIS: the line passing through the foci.

PARTS OF AN ELLIPSE

VERTICES: the intersection of the principal axis and the ellipse, denoted by V.

PARTS OF AN ELLIPSE

MAJOR AXIS: the line segment whose endpoints are the vertices.

PARTS OF AN ELLIPSE

CENTER: the midpoint of the major axis, denoted by C.

PARTS OF AN ELLIPSE

MINOR AXIS: the line segment perpendicular to the major axis at the center and whose endpoints are on the ellipse.

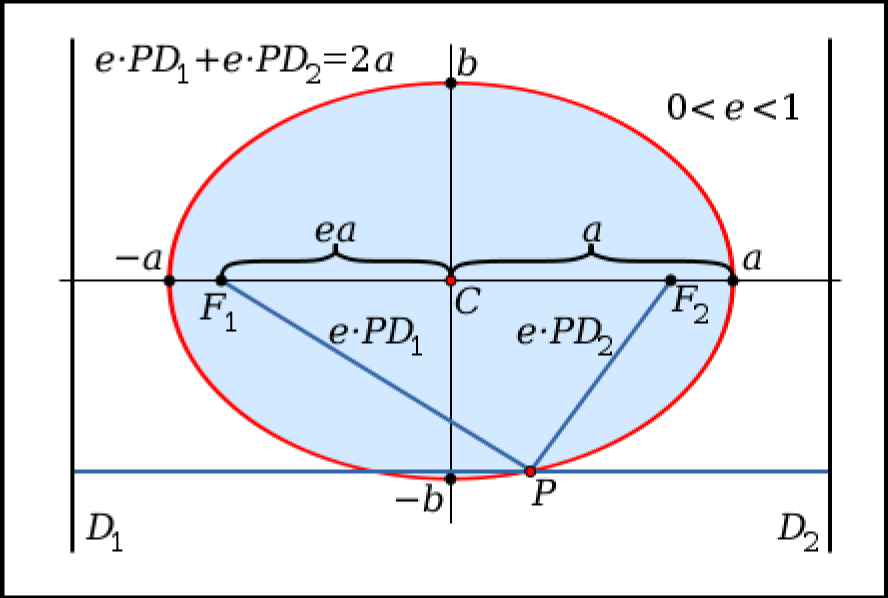
PARTS OF AN ELLIPSE

EXTREMITIES: the endpoints of minor axis, denoted by B.

PARTS OF AN ELLIPSE

DIRECTRICES: the fixed lines in the analytic definition of a general conic, denoted by D.

EQUATION OF THE ELLIPSE



Let CV = a, CB = b, CF = c, CD = d

EQUATION OF THE ELLIPSE

Then, for a horizontal ellipse with C(0, 0), we have

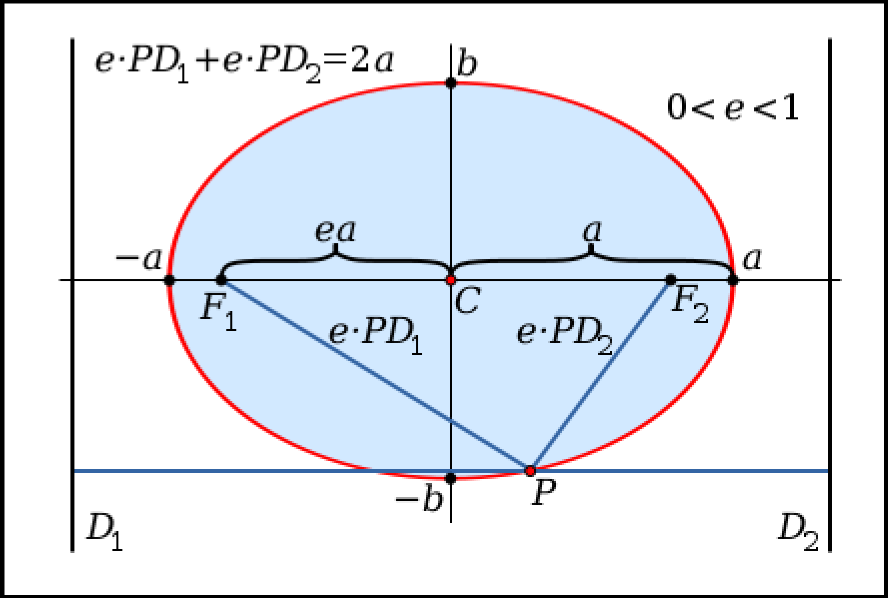
V(a, 0), (-a, 0)

B(0, b), (0, -b)

F(c, 0), (-c, 0)

D: x = d, x = -d

EQUATION OF THE ELLIPSE



From definition:

$$PF_1 + PF_2 = k$$

EQUATION OF THE ELLIPSE

Using V_1 as P:

$$V_1F_1 + V_1F_2 = k$$

$$(a - c) + (a + c) = k$$

$$2a = k$$

Hence,

$$PF_1 + PF_2 = 2a$$

EQUATION OF THE ELLIPSE

Using B_1 as P:

$$B_1F_1 + B_1F_2 = 2a$$

But $B_1F_1 = B_1F_2$ by symmetry, thus

$$2B_1F_1 = 2a$$

$$B_1F_1 = a$$

Hence,

$$b^2 = a^2 - c^2$$

EQUATION OF THE ELLIPSE

$$\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=2a$$

$$\sqrt{(x+c)^2+y^2}=2a-\sqrt{(x-c)^2+y^2}$$

$$(x+c)^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+(x-c)^2+y^2$$

$$2xc=4a^2-4a\sqrt{(x-c)^2+y^2}-2xc$$

$$4a\sqrt{(x-c)^2+y^2}=4a^2-4xc$$

EQUATION OF THE ELLIPSE

$$\sqrt{(x-c)^2+y^2}=a-\frac{xc}{a}$$

$$x^2-2xc+c^2+y^2=a^2-2xc+\frac{x^2c^2}{a^2}$$

$$x^2+c^2+y^2=a^2+\frac{x^2c^2}{a^2}$$

$$x^2\left(1-\frac{c^2}{a^2}\right)+y^2=a^2-c^2$$

EQUATION OF THE ELLIPSE

$$x^2\left(1-\frac{c^2}{a^2}\right)+y^2=a^2-c^2$$

$$x^2\left(\frac{a^2-c^2}{a^2}\right)+y^2=a^2-c^2$$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1$$

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$$

EQUATION OF THE ELLIPSE

With C:(o, o)

Horizontal Ellipse:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Vertical Ellipse:

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

EQUATION OF THE ELLIPSE

With C:(h, k)

Horizontal Ellipse:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Vertical Ellipse:

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

EXAMPLE 1:

Find the equation of the ellipse with center (0, 0), a vertex at (5, 0) and an extremity at (0, 4).

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

ECCENTRICITY OF THE ELLIPSE

Recall: $$e = \frac{PF}{PD}$$

Using B_1 as P, $$e = \frac{B_1F}{B_1D} = \frac{a}{d}$$ EQ1

Using V_1 as P, $$e = \frac{V_1F}{V_1D} = \frac{a-c}{d-a}$$ EQ2

Solving EQ1&2, we get $$e = \frac{c}{a}$$

FORMULAS RELATING a, b, c, d, & e

$$c^2 = a^2 - b^2$$

$$e = \frac{c}{a}$$

$$d = \frac{a}{e} = \frac{a^2}{c}$$

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, and ecentricity of the ellipse

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

C:(o, o)

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, and ecentricity of the ellipse

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

V:(0, 5),(0, -5)

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, and ecentricity of the ellipse

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

B:(3, 0),(-3, 0)

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, and ecentricity of the ellipse

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

F:(0, 4),(0, -4)

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, and ecentricity of the ellipse

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

D: y = 25/4, y = -25/4

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, and ecentricity of the ellipse

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

e = 4/5

EXAMPLE 3:

A point moves along the cartesian plane so that its distance from the point (1, 0) is ½ of its distance from the line x = 4. What is the equation of the path of the point?

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

EXAMPLE 4:

What is the equation of the ellipse with a vertex at (3, -2), a corresponding focus at (0, -2) and a corresponding directrix at (8, -2)?

$$\frac{\left(x+\frac{9}{2}\right)^2}{\frac{225}{4}}+\frac{\left(y+2\right)^2}{36}=1$$

EXAMPLE 5:

Find the center, vertices, extremities, foci, directrices, and eccentricity of the ellipse

$4x^2 + 9y^2 - 16x + 18y - 11 = 0$

C:(2, -1)

EXAMPLE 5:

Find the center, vertices, extremities, foci, directrices, and eccentricity of the ellipse

$$4x^2 + 9y^2 - 16x + 18y - 11 = 0$$

V:(5, -1), (-1, -1)

EXAMPLE 5:

Find the center, vertices, extremities, foci, directrices, and eccentricity of the ellipse

$4x^2 + 9y^2 - 16x + 18y - 11 = 0$

B:(2, 1), (2, -3)

EXAMPLE 5:

Find the center, vertices, extremities, foci, directrices, and eccentricity of the ellipse

$$4x^2 + 9y^2 - 16x + 18y - 11 = 0$$

$$F: \left(2+\sqrt{5}, -1\right), \left(2-\sqrt{5}, -1\right)$$

EXAMPLE 5:

Find the center, vertices, extremities, foci, directrices, and eccentricity of the ellipse

$$4x^2 + 9y^2 - 16x + 18y - 11 = 0$$

$$D: x = 2 + \frac{9\sqrt{5}}{5}, x = 2 - \frac{9\sqrt{5}}{5}$$

EXAMPLE 5:

Find the center, vertices, extremities, foci, directrices, and eccentricity of the ellipse

$$4x^2 + 9y^2 - 16x + 18y - 11 = 0$$

$$e = \frac{\sqrt{5}}{3}$$

EXAMPLE 6:

A whispering gallery has a semi-elliptic arch which is 25 ft high at the middle and 5 ft high at the ends. If the whispering points are 30 ft apart, how high is the ceiling (from the ground) above the whispering points?

21 ft

HOMEWORK#2:

TCWAG6

Section 10.2 Exercises

#s 10, 20, 26, 32, 34