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MATH 5

HYPERBOLA

OBJECTIVES:

- derive the standard equation of a hyperbola
- use the equation of a hyperbola to determine its properties
- find the equation of a hyperbola given some of its properties
- express the equation of a hyperbola in both the standard and general forms
- solve problems using the equation of a hyperbola

ANALYTIC DEFINITION

HYPERBOLA: set of all points in a plane the absolute value of the difference of whose distances from two fixed points is a constant.

PARTS OF A HYPERBOLA

FOCI: the two fixed points in the analytic definition, denoted by F.

PARTS OF A HYPERBOLA

PRINCIPAL AXIS: the line passing through the foci.

PARTS OF A HYPERBOLA

VERTICES: the intersection of the principal axis and the hyperbola, denoted by V.

PARTS OF A HYPERBOLA

TRANSVERSE AXIS: the line segment whose endpoints are the vertices.

PARTS OF A HYPERBOLA

CENTER: the midpoint of the transverse axis, denoted by C.

PARTS OF A HYPERBOLA

ASYMPTOTES: the lines to which the graph of the hyperbola approach.

PARTS OF A HYPERBOLA

AUXILIARY RECTANGLE: the rectangle whose vertices lie on the asymptotes of the hyperbola and whose sides contain the vertices of the hyperbola.

PARTS OF A HYPERBOLA

CONJUGATE AXIS: the line segment perpendicular to the transverse axis at the center and whose endpoints are on the auxiliary rectangle.

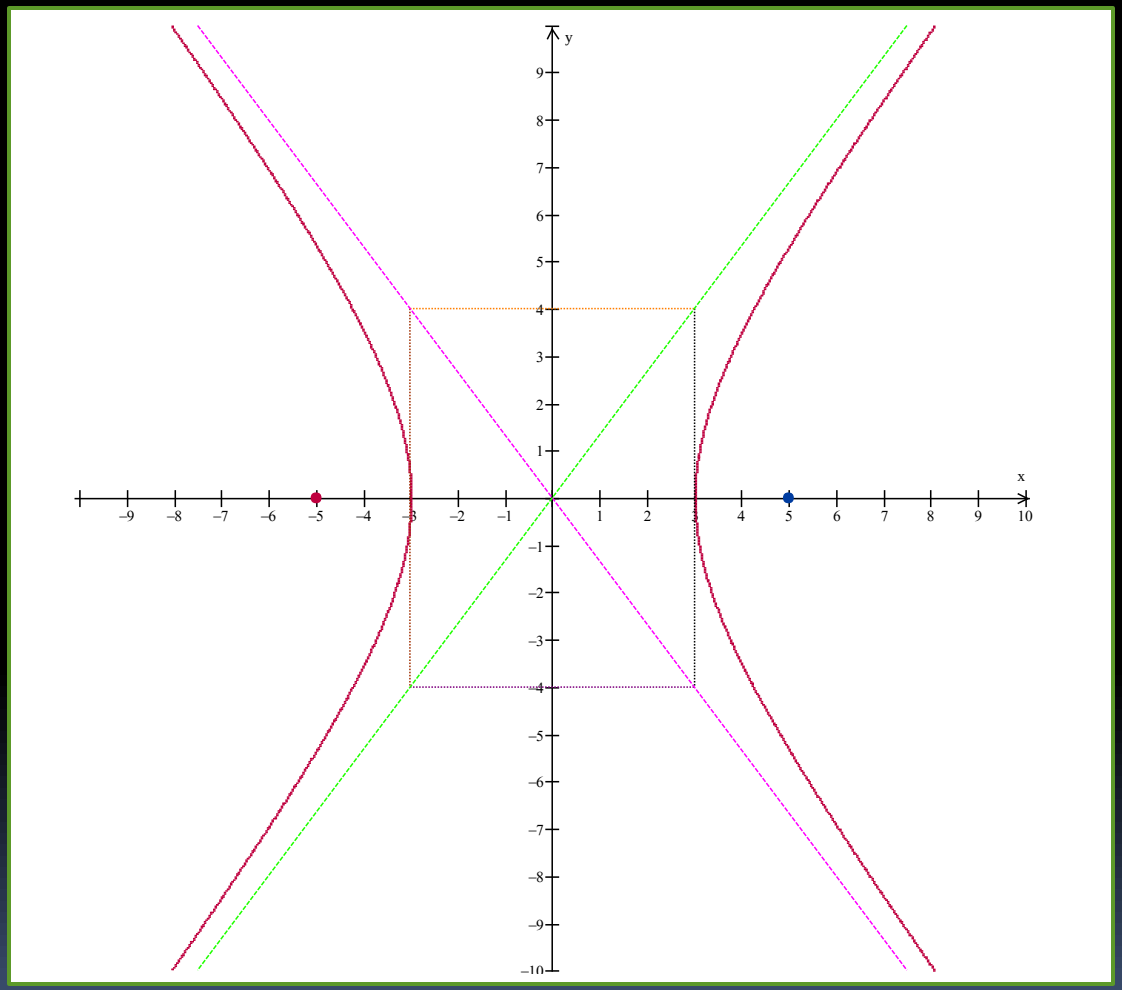
PARTS OF A HYPERBOLA

EXTREMITIES: the endpoints of conjugate axis axis, denoted by B.

PARTS OF A HYPERBOLA

DIRECTRICES: the fixed lines in the analytic definition of a general conic, denoted by D.

EQUATION OF THE HYPERBOLA



EQUATION OF THE HYPERBOLA

Let CV = a, CB = b, CF = c, CD = d

Then, for a horizontal hyperbola with C(o, o), we have

V(a, o), (-a, o)

B(o, b), (o, -b)

F(c, o), (-c, o)

D: x = d, x = -d

EQUATION OF THE HYPERBOLA

From definition:

$$|PF_1 - PF_2| = k$$

Using V_1 as P:

$$|V_1F_1 - V_1F_2| = k$$

$$|(c - a) - (c + a)| = k$$

$$2a = k$$

Hence, $|PF_1 - PF_2| = 2a$

EQUATION OF THE HYPERBOLA

$$\sqrt{(x-c)^2+y^2}-\sqrt{(x+c)^2+y^2}=\pm 2a$$

$$-\sqrt{(x+c)^2+y^2}=\pm 2a-\sqrt{(x-c)^2+y^2}$$

$$(x+c)^2+y^2=4a^2\mp 4a\sqrt{(x-c)^2+y^2}+x(-c)^2+y^2$$

$$2xc=4a^2\mp 4a\sqrt{(x-c)^2+y^2}-2xc$$

$$\pm 4a\sqrt{(x-c)^2+y^2}=4a^2-4xc$$

EQUATION OF THE HYPERBOLA

$$\pm\sqrt{(x-c)^2+y^2}=a-\frac{xc}{a}$$

$$x^2-2xc+c^2+y^2=a^2-2xc+\frac{x^2c^2}{a^2}$$

$$x^2+c^2+y^2=a^2+\frac{x^2c^2}{a^2}$$

$$x^2\left(1-\frac{c^2}{a^2}\right)+y^2=a^2-c^2$$

EQUATION OF THE HYPERBOLA

$$x^2\left(1-\frac{c^2}{a^2}\right)+y^2=a^2-c^2$$

$$x^2\left(\frac{a^2-c^2}{a^2}\right)+y^2=a^2-c^2$$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, \quad \text{where} \quad b^2=c^2-a^2$$

EQUATION OF THE HYPERBOLA

With C:(o, o)

Horizontal Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertical Hyperbola : $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

EQUATION OF THE HYPERBOLA

With C:(h, k)

Horizontal Hyperbola :

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Vertical Hyperbola :

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

EXAMPLE 1:

Find the equation of the hyperbola with center (0, 0), a vertex at (5, 0) and an extremity at (0, 4).

$$\frac{x^2}{25} - \frac{y^2}{16} = 1$$

FORMULAS RELATING a, b, c, d, & e

$$c^2 = a^2 + b^2$$

$$e = \frac{c}{a}$$

$$d = \frac{a}{e} = \frac{a^2}{c}$$

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, asymptotes, and eccentricity of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

C:(0, 0)

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, asymptotes, and eccentricity of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

V:(3, 0),(-3, 0)

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, asymptotes, and eccentricity of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

B:(0, 4),(0, -4)

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, asymptotes, and eccentricity of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

F:(5, 0),(-5, 0)

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, asymptotes, and eccentricity of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

D: $x = 9/5$, $x = -9/5$

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, asymptotes, and eccentricity of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

A: y = (4/3) x, y = (-4/3) x

EXAMPLE 2:

Find the center, vertices, extremities, foci, directrices, asymptotes, and eccentricity of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

$e = 5/3$

EXAMPLE 3:

A point moves along the cartesian plane so that its distance from the point (4, 0) is twice its distance from the line x = 1. What is the equation of the path of the point?

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

EXAMPLE 4:

What is the equation of the hyperbola with a vertex at (5, -2), a corresponding focus at (0, -2) and a corresponding directrix the line x = 8?

$$\frac{\left(x-\frac{25}{2}\right)^2}{\frac{225}{4}}-\frac{(y+2)^2}{100}=1$$

EXAMPLE 5:

Express the following equation of the hyperbola in standard form:

$4x^2 - 9y^2 - 16x - 18y + 43 = 0$

$$\frac{(y+1)^2}{4} - \frac{(x-2)^2}{9} = 1$$

EXAMPLE 6:

Find an equation of the hyperbola whose foci are the vertices of the ellipse $7x^2 + 11y^2 = 77$ and whose vertices are the foci of this ellipse.

$$\frac{x^2}{4} - \frac{y^2}{7} = 1$$

HOMEWORK#3:

TCWAG6

Section 10.3 Exercises

#s 10, 20, 26, 32, 33